

# MULTIPLE-USER NETWORKS PERFORMANCE ANALYSIS AND RELAY SELECTION

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## ABSTRACT

*This paper examines the relay selection (RS) problem in networks with multiple users and multiple common amplify-and-forward (AF) relays. Further it gives behavioural aspects of RS in multiple user relay networks. Optimal RS (ORS) algorithm is an extension of RS maximizes the minimum end-to-end receive signal-to-noise ratio (SNR) of all users. The convolution of ORS is equilateral to both number of relays and users. There is also an introduction to suboptimal RS scheme which has minimal complications in number of relays and equivalent complications in*

*number of users. Moreover various orders of ORS and SRS are compared with simple RS scheme and single user keys. The ORS is shown to achieve full diversity, while the diversity order of the SRS decreases with the number of users. The benefits of RS scheme over SRS increases with number of relays in the network. Analytical results are calculated with the help of simulations.*

**Index Terms** - Relay selection Array gain, diversity order, multiple-user networks, outage probability.

## 1.INTRODUCTION

Cooperative communication is a communication which allows communication terminals in a network to hear and help the information transmission of each other, by taking advantage of the broadcast nature of wireless communication. Here RS is the essential technique for cooperative technique because properly designed RS can attain full structural diversity with little effort and overhead. RS issues have been read up on open writings for networks with single user networks, e.g., relay networks have large interests with multiple user network. Multiple-user networks include ad-hoc, sensor, and mesh networks. As, RS schemes offered for single-user networks shall not be expanded to multiple-user networks. The relays are sorted out on basis of the power of the user's channels to the relays.

In this testing, we consider a multiple-user multiple-relay network where each user can only be helped by a single relay and one relay can help at most one user. There are some benefactions of this paper listed below.

Multiplicity organization of the ORS and SRS are examine thoughtfully using order statistics.  $N$  user networks,  $Nr (\geq N)$  relays, and no straight links, for the ORS, all users have multiplicity order  $Nr$ , which is the full multiplicity order of a single-user network with  $Nr$  relays. Thus, relays for user rivalry does not influence the multiplicity order if optimally designed.  $Nr - N + 1$  is shown the multiplicity order of all users for SRS. The users have multiplicity order  $Nr + 1$  and  $\max(Nr - N + 2, 1)$  for the ORS and the SRS, only when there is straight links in the network.

Compact higher bound on the blackout expectations of the ORS and SRS are derived. It is shown systematically that the SRS carry off better array gain than a naive RS, and its edge increases as the number of relays increases.

Parallel blackout expectations are printed to explain our systematic results and differentiate the ORS, the SRS, and a naive RS.

The remaining paper is arranged as follows.

## II. SYSTEM MODEL AND ORDER STATISTICS

### III. RS SCHEMES

### IV. DIVERSITY ORDER ANALYSIS

### V. OUTAGE PROBABILITY ANALYSIS FOR TWO-USER NETWORKS

### VI. RELAY SELECTION IN NETWORKS WITH DIRECT LINKS

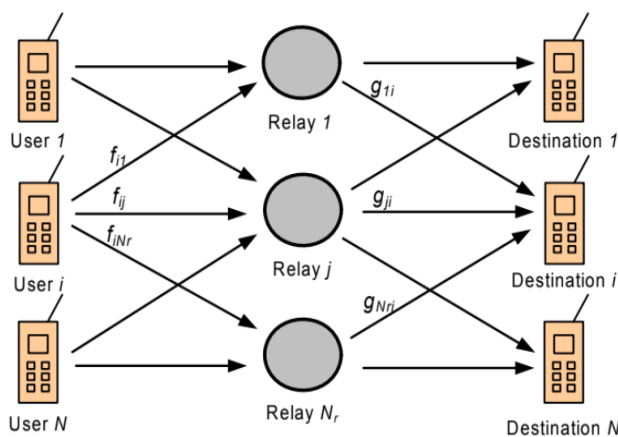


Fig. 1. A multiple-user multiple-relay network model.

## II. SYSTEM MODEL AND ORDER STATISTICS

### A. System Model

Fig 1. Shows the wireless relay network by considering  $N$  users sending information to their station via  $N_r$  relay nodes. It also shows that each node has single antenna and  $P$  the power budget while each relay is denoted by  $Q$ . The fading coefficients from the  $i$ th user to the  $j$ th relay and from the  $j$ th relay to the  $i$ th destination are denoted as  $f_{ij}$  and  $g_{ji}$ , respectively. Between the users and destinations there are no straight links. Channels are assumed to be independent and identically distributed (i.i.d.) complex Gaussian with zero-mean and unit variance, i.e.,  $f_{ij}, g_{ji} \sim \text{CN}(0, 1)$ . The channel amplitudes,  $|f_{ij}|$  and  $|g_{ji}|$ , thus follow the Rayleigh distribution.

To send the information users need relays help. We suppose that each user will be supported by one and only one relay and it minimizes the synchronization requirement on the network.

The transferral from the users to the relays, is the first phase and the transmission from the relays to the destinations, is the second phase. To circumvent involvement, the users are allocated orthogonal channels using frequency-division or time-division multiple access. Without loss of generality, the transmission of User  $i$  helped by Relay  $j$  is elaborated here. Indicate the information symbol of User  $i$  as  $x_i$ , which has unit average energy.

$$Y_{ij} = \sqrt{\frac{PQ}{P|f_{ij}|^2+1}} f_{ij} g_{ji} x_i + \sqrt{\frac{Q}{P|f_{ij}|^2+1}} g_{ji} n_{rj} + n_{di} ,$$

where  $n_{rj}$  and  $n_{di}$  are the extra noises at Relay  $j$  and Destination  $i$ , respectively, which are supposed to be i.i.d. following CN (0, 1). The end-to-end receive SNR, or SNR in short, of User  $i$  thus equals

$$Y_{ij} = \frac{PQ|f_{ij}g_{ji}|^2}{P|f_{ij}|^2+Q|g_{ji}|^2+1}$$

### B. Performance Measure

In favour execution count for cooperative networks are blackout chance, diversity order, and array gain. For a user, an blackout occurs if the user SNR drops below a prearranged SNR threshold  $\gamma_{th}$ . Denote the blackout chance communicate to  $\gamma_{min}$  (the worst SNR among all users) as  $P_{out,upp}$ . For all users, since their SNRs are always not lower than  $\gamma_{min}$ , their blackout possibilities are upper bounded by  $P_{out,upp}$ .

### C. SNR Distribution and Order Statistics

In this segment, we analyse results on the SNR distribution and order statistics to be used for theoretical analysis later.

$$F_{\gamma}(x) = 1 - 2 \sqrt{\frac{x(x+1)}{PQ}} e^{-\left(\frac{1}{P} + \frac{1}{Q}\right)x} K_1\left(2 \sqrt{\frac{x(x+1)}{PQ}}\right)$$

$F_{\gamma}(x)$  can be well-approximated as:

$$F_{\gamma}(x) \approx 1 - e^{-\left(\frac{1}{P} + \frac{1}{Q}\right)x} = \left(\frac{1}{P} + \frac{1}{Q}\right)x \sum_{i=2}^{\infty} \frac{1}{i!} \left[-\left(\frac{1}{P} + \frac{1}{Q}\right)x\right]^i$$

## III. RS SCHEMES

An RS scheme is grown, in which a “linear marking” device is used to maximize the minimal SNR among the users. Initially practical relay node assignment is randomly selected, by which each user target pair interference with the support of a relay node. In a number of iterations the relay assignment algorithm is then adjusted. During each repetition, the user that has the slightest SNR, shown as  $\gamma_{min}$ , finds a finer relay such that its SNR can be grown. If the finer relay has been given

to another user User  $a$  tries to switch to another relay under the circumstances that the resulted SNR is bigger than  $\gamma_{\min}$ . If Relay  $b$  given to other user, additional accommodation to that user's relay assignment is required. Therefore, relay adjustment with the minimal SNR, of the user, may have a series effect on the relay assignment of multiple users. If such adjustments are there in presence, the minimum SNR of the users increases and scheme moves to other repetition. RS schemes worst case complexity is  $O(N N_r^2)$ . Therefore, the total complexity for the SRS is,

$$C = \sum_{l=1}^N [(N_r - l)(N - l + 1) + (N - l)]$$

## IV. DIVERSITY ORDER ANALYSIS

### A. Diversity Order of ORS

A. Lemma 1: Along With the ORS, the worst case is  $\gamma_{\min} = \gamma(N-1) N_r + 1$ . And a enough and required situation for the worst case is: (a) the smallest  $N_r$  elements of  $\Gamma$ , i.e.,  $\gamma(N-1) N_r + 1$ ,  $\gamma(N-1) N_r + 2, \dots, \gamma N N_r$ , are all in the same row<sup>3</sup>, when  $N < N_r$ ; or (b) the smallest  $N_r$  elements of  $\Gamma$  are either in the same row or in the same column<sup>4</sup>, when  $N = N_r$ .

Proof: We first prove  $\gamma_{\min} \geq \gamma(N-1) N_r + 1$  with the ORS by using proof by conflict. Consider that for a given channel awareness, we have  $\gamma_{\min} < \gamma(N-1) N_r + 1$  in the ORS result. Then in  $\Gamma$ , the number of elements smaller than  $\gamma_{\min}$  is less than or equal to  $N_r - 2$ . With profit of majority, assume that  $\gamma_{\min}$  is situated in the  $i^*$ th row and the  $j^*$ th column of  $\Gamma$ , i.e.,  $\gamma_{\min} = \gamma_{i^*j^*}$ . Let  $R <$  denote the set of row indices of the elements (in  $\Gamma$ ) that are smaller than  $\gamma_{\min}$  and are located in the  $j^*$  column. Let  $C \leq$  denote the set of indices of the columns in which  $\gamma_{\min}$  and all elements (in  $\Gamma$ ) smaller than  $\gamma_{\min}$  are located. Then we have  $|C \leq| \leq (N_r - 2) - |R <| + 1$ , where  $|\cdot|$  means the cardinality of a set. So that, for the set  $C \leq = \{1, 2, \dots, N_r\} \setminus C \leq$ , we have  $|C \leq| \geq N_r - ((N_r - 2) - |R <| + 1) = |R <| + 1$ . Note that all elements in any column in  $C \leq$  are larger than  $\gamma_{\min}$ . Since  $|C \leq| > |R <|$ , there is a presence of a column index denoted  $j^\dagger \in C \leq$  such that in the ORS answers for the given channel awareness, Relay  $j^\dagger$  is either not gives to any user or is gives to a user, denoted User  $i^\dagger$ , satisfying  $i^\dagger \in R / <$ . If Relay  $j^\dagger$  is not assigned to any user, we change User  $i^*$  from Relay  $j^*$  to Relay  $j^\dagger$ , which gives User  $i^*$  an SNR larger than  $\gamma_{\min}$ . If Relay  $j^\dagger$  is assigned to User  $i^\dagger$ , we switch the relay assignment for Users  $i^*$  and  $i^\dagger$ , and after the switching, both users have SNRs larger than  $\gamma_{\min}$ . In either case, the new RS result has a minimum user SNR larger than  $\gamma_{\min}$ . This conflicts that the ORS maximizes the minimal SNR of the users.

The sufficiency of (a) and (b) is straightforward. Thus, it

can be ended that the worst case with the ORS is  $\gamma_{\min} = \gamma(N-1) N_r + 1$ .

B. Diversity Order of SRS

Theorem 2: With the SRS, the diversity order of each user is  $N_r - N + 1$ .

Proof: Similar as the proof of Theorem 1, the best case is  $\gamma_{min} = \gamma_N$ . The worst case happens when the abiding  $N_r - (N - 1)$  elements of  $\Gamma$  for the last user in the RS are the smallest  $N_r - (N - 1)$  elements. In this situation,  $\gamma_{min} = \gamma_{(N-1)(N_r+1)+1}$ . So  $\gamma_{min}$  can take

$\gamma_N, \gamma_{N+1}, \dots,$  or  $\gamma_{(N-1)(N_r+1)+1}$ . The blackout possibility communicating to  $\gamma_{min}$  can be calculated as  $P_{out, upp, SRS} = (N-1)(N_r+1)+1$

$k=N$  Prob ( $\gamma_{min} = \gamma_k$ )  $F_{\gamma k}(\gamma_{th})$ . Same as the

proof of Theorem 1, for any  $k$ , Prob ( $\gamma_{min} = \gamma_k$ ) does not

turn on  $P$ . When  $Q = P \max \{\gamma_{th}, 1\}$ , we have  $P_{out, upp, SRS} \approx$  Prob

$\Gamma_{min} = \gamma_{(N-1)(N_r+1)+1} (2\gamma_{th})$

$N_r - N + 1 (N N_r)!$

$(N_r - N + 1)(N_r - N)! ((N - 1)(N_r + 1)) \times P^{-(N_r - N + 1)} + O P^{-(N_r - N + 2)}$

This reveals that an attainable diversity order of every user is not more than  $N_r - N + 1$ . For a specific user, say User  $K$ , same as the proof of Theorem 1, the outage probability can be lower bounded as  $P_{out, User-K, SRS} \geq 1$

$N P_{out, upp, SRS} \sim O P^{-(N_r - N + 1)}$ . This means that User  $K$  has diversity order not less than  $N_r - N + 1$ . Along with the actual reality that each user has diversity

order not less than  $N_r - N + 1$ , it can be ended that each

user has diversity order  $N_r - N + 1$ .

V. OUTAGE PROBABILITY ANALYSIS FOR TWO-USER NETWORKS

A. Outage Probability Bound of ORS

Theorem 3: For a two-user network, with the ORS, the blackout possibilities of both users are upper surrounded by

$$P_{out, upp, ORS} = \frac{N_r - 1}{2N_r - 1} F_{\gamma 2}(\gamma_{th}) + \frac{N_r + 2}{2(2N_r - 1)} F_{\gamma 3}(\gamma_{th}) + \sum_{i=4}^{N_r + 1} \frac{2N_r(N_r)}{(2N_r - (i - 1))(2N_r)} F_{\gamma i}(\gamma_{th}),$$

Proof:  $\gamma_{\min}$  can take  $\gamma_2, \gamma_3, \dots$ , or  $\gamma_{Nr+1}$ . The blackout probability upper bound,  $P_{out, upp}$ , ORS, can be calculated as

$$\begin{aligned} P_{out, upp, ORS} &= \text{Prob}(\gamma_{\min} < \gamma_{th}) \\ &= \sum_{k=2}^K \text{Prob}(Y_{min} = Y_k) \text{Prob}(\gamma_k \leq \gamma_{th}) \\ &= \sum_{k=2}^K \text{Prob}(\gamma_{min} = \gamma_k) F_{\gamma_k}(\gamma_{th}) \end{aligned}$$

#### A. Outage Probability Bound of SRS

Theorem 4: For a two-user network, with the SRS, the blackout possibilities of two users in the network are upper surrounded by

$$P_{out, upp, SRS} = \frac{N_r-1}{2N_r-1} F_{\gamma_2}(\gamma_{th}) + \frac{N_r+1}{2(2N_r-1)} F_{\gamma_3}(\gamma_{th}) + \sum_{i=4}^{N_r+1} \frac{2N_r(N_r-k-1)}{(2N_r-(k-1))(2N_r-k-1)} F_{\gamma_i}(\gamma_{th})$$

Proof: With the SRS,  $\gamma_{\min}$  can take  $\gamma_2, \gamma_3, \dots$ , or  $\gamma_{Nr+2}$ .

## VI. RELAY SELECTION IN NETWORKS WITH DIRECT LINKS

#### A. System Model and RS Schemes

We assume that each and every relay can help at most one user, and a user can be supported by at most one relay. The condition  $N_r \geq N$  is not needed in this situation due to the straight links.

$$y_{id} = \sqrt{P} h_{idxi} + n_{di}$$

where  $n_{di}$  is the extra noise at goal  $i$  which follows CN  $(0, 1)$  and  $h_{id}$  gives the channel of the direct link for User  $i$  which follows CN  $(0, 1)$ . The receive SNR of User  $i$  via the direct link equals  $\gamma_{id} = P|h_{id}|^2$ . The CDF of  $\gamma_{id}$ ,  $F_{\gamma_{id}}(x)$ , is

$$F_{\gamma_{id}}(x) = 1 - e^{-\frac{x}{p}} = \frac{x}{p} - \sum_{j=2}^{\infty} \frac{(-1)^j x^j}{j! P^j}.$$

#### B. Diversity Order Analysis

Theorem 5: For a multiple-user network with straight links, with the ORS, each user has diversity order  $N_r + 1$ ; with the SRS, each user has diversity order  $\max(N_r - N + 2, 1)$ .

Proof: We arrange the nonzero entries of  $\Gamma$ ,  $\gamma_{ij}$ 's and  $\gamma_{id}$ 's, in falling order as  $\gamma_1 > \dots > \gamma_k > \dots > \gamma_{N(N_r+1)}$  where  $\gamma_k$  is the  $k$ th largest element of  $\Gamma$ .

## VII. NUMERICAL AND SIMULATION RESULTS

In this part, we show duplicate results to explain our analysis, and to elaborate the performance of the ORS, SRS, and naive RS schemes. All nodes are assumed to have the same power, i.e.,  $Q = P$ .

The SNR threshold  $\gamma_{th}$  is set to be 0 dB. Firstly we confirm the obtained diversity orders of various RS schemes. Fig. 2 shows the duplicate blackout probabilities of the ORS, the SRS, the naive RS schemes, and a random RS scheme in a three-user network with four relays. In the alternative

RS, each user alternatively chooses a relay without dispute. For the ORS, the SRS, and the random RS scheme, due to the uniformity of the network, User 2 and User 3 have the same blackout probability as User 1, and thus, only the outage probability of User 1 is shown. For the naive RS scheme, outage probabilities of the three users are various. User 1 achieves the presentation of the single-user case since it has all  $N_r$  relays to choose from. User 2 has worse presentation than User 1, but has good performance than User 3. In Fig. 2, reference lines (dashed lines) with slopes 1, 2, 3, and 4 are also two and four relays. In Fig. 4, with two and four relays, for two-user networks, we present the duplicate blackout possibilities of User 1 with the ORS and SRS schemes and differentiate with the outage probability upper bounds derived using the minimum SNR,

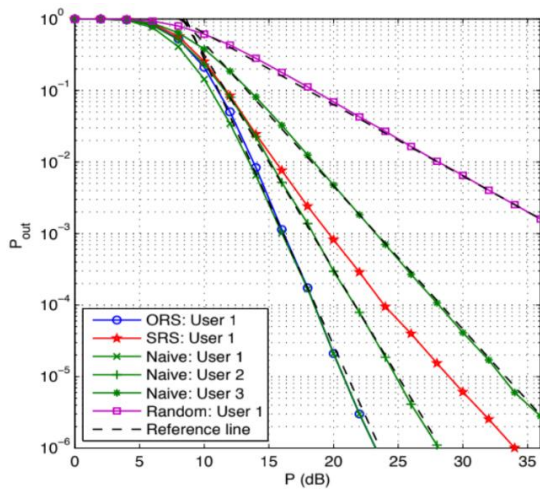


Fig. 2. Outage probabilities for a network with three users and four relays for the ORS, SRS, naive, and random RS.

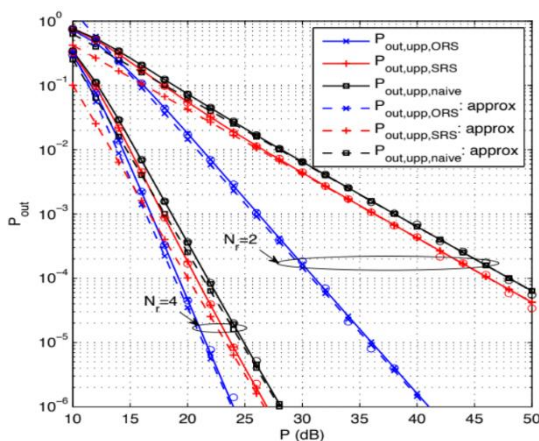


Fig. 3. Outage probabilities corresponding to  $\gamma_{min}$  for networks with two users and two or four relays for the ORS, SRS, and naive RS.

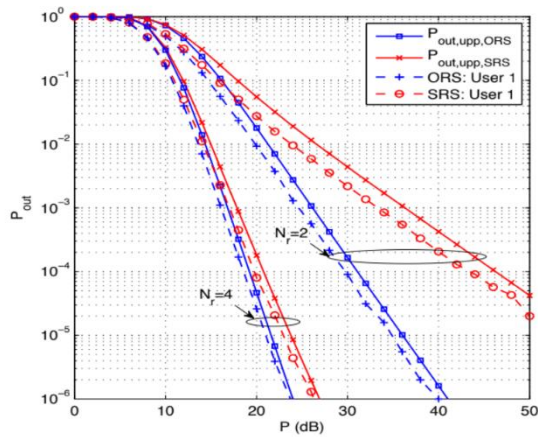


Fig. 4. Outage probabilities corresponding to  $\gamma_{min}$  and of users in network with two users and two or four relays for the ORS and SRS.

We additionally explore the array gain differences (1) int between the ORS and the single-user best-relay situation and (2) between the SRS and the naive RS. Fig. 5 shows duplicate results of the outage probability bounds of the ORS, SRS, and naive RS ( $P_{out, upp, ORS}$ ,  $P_{out, upp, SRS}$ ,  $P_{out, upp, naive}$ ) and the outage probability in the naive RS scheme of User 1. In Fig. 6, presentation of the duplicate user outage probabilities of the ORS and SRS for a three-user network with straight links. We assume two conditions:  $N_r = 2$  (for the case  $N > N_r$ ) and  $N_r = 3$  (for the case  $N_r \geq N$ ). Reference lines (dashed lines) with slopes 1, 2, 3, and 4 are also drawn.

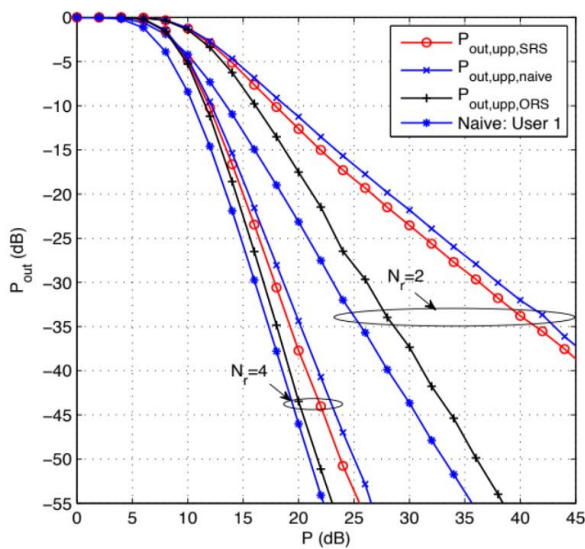


Fig. 5. Array gain difference (observed from outage probabilities) between the ORS and single-user best-relay case, and between the SRS and naive RS.



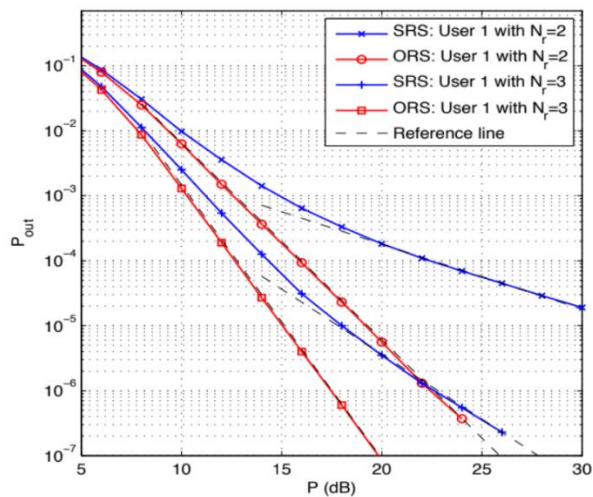


Fig. 6. Outage probabilities for a network with direct links, three users and two or three relays for the ORS and SRS.

## VIII. CONCLUSION

In this paper, research of relay selection problem in network is done. There is also in presence a suboptimal relay scheme, whose complexity is quadratic in number of relays. For performance differentiation, naïve scheme is also derived. The suboptimal relay selection achieves a higher array gain than the naive relay selection.

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